

Topic 8

BC only

Power Series

2000 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

3. The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 5$ is given by $f^{(n)}(5) = \frac{(-1)^n n!}{2^n (n+2)}$, and $f(5) = \frac{1}{2}$.
- (a) Write the third-degree Taylor polynomial for f about $x = 5$.
 - (b) Find the radius of convergence of the Taylor series for f about $x = 5$.
 - (c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\frac{1}{1000}$.

END OF PART A OF SECTION II

2001 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

- (a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim_{x \rightarrow 0} \frac{f(x) - \frac{1}{3}}{x}$.

- (c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x) dx$.

- (d) Find the sum of the series determined in part (c).
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END OF EXAMINATION

2002 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

- (a) Find the interval of convergence of the Maclaurin series for f . Justify your answer.
 - (b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.
 - (c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.
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2003 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

for all real numbers x .

- (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$.
Give a reason for your answer.
- (b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.
- (c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.
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END OF EXAMINATION

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2004 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.
- (a) Find $P(x)$.
- (b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.
- (c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$.
- (d) Let G be the function given by $G(x) = \int_0^x f(t) dt$. Write the third-degree Taylor polynomial for G about $x = 0$.
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END OF EXAMINATION

2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the n th derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the n th derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.
- (a) Write the sixth-degree Taylor polynomial for f about $x = 2$.
- (b) In the Taylor series for f about $x = 2$, what is the coefficient of $(x - 2)^{2n}$ for $n \geq 1$?
- (c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.
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2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.
- (a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.
- (b) Is it possible for the x -axis to be tangent to the graph of f at some point? Explain why or why not.
- (c) Find the second-degree Taylor polynomial for f about $x = -1$.
- (d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

2006 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.

- (a) Find the interval of convergence of the power series for f . Justify your answer.
- (b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

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2007 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. Let f be the function given by $f(x) = e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Use your answer to part (a) to find $\lim_{x \rightarrow 0} \frac{1 - x^2 - f(x)}{x^4}$.

(c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2} dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2} dt$.

(d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2} dt$ by less than $\frac{1}{200}$.

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END OF EXAM

2008 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

x	$h(x)$	$h'(x)$	$h''(x)$	$h'''(x)$	$h^{(4)}(x)$
1	11	30	42	99	18
2	80	128	$\frac{488}{3}$	$\frac{448}{3}$	$\frac{584}{9}$
3	317	$\frac{753}{2}$	$\frac{1383}{4}$	$\frac{3483}{16}$	$\frac{1125}{16}$

3. Let h be a function having derivatives of all orders for $x > 0$. Selected values of h and its first four derivatives are indicated in the table above. The function h and these four derivatives are increasing on the interval $1 \leq x \leq 3$.
- (a) Write the first-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$. Is this approximation greater than or less than $h(1.9)$? Explain your reasoning.
- (b) Write the third-degree Taylor polynomial for h about $x = 2$ and use it to approximate $h(1.9)$.
- (c) Use the Lagrange error bound to show that the third-degree Taylor polynomial for h about $x = 2$ approximates $h(1.9)$ with error less than 3×10^{-4} .
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

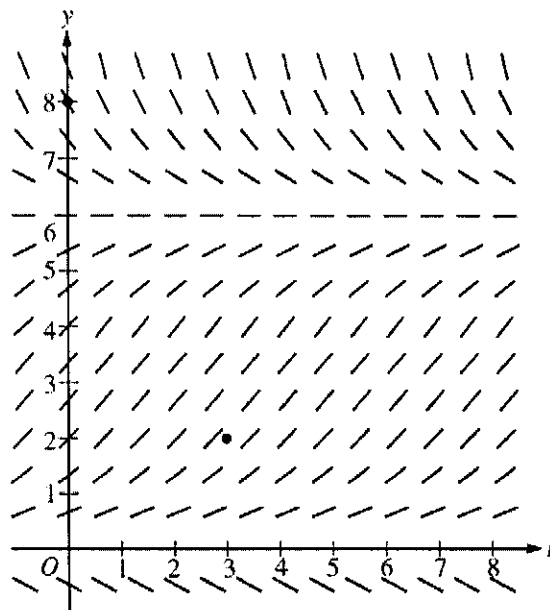
END OF PART A OF SECTION II

2008 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

- (a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.

(Note: Use the axes provided in the exam booklet.)



- (b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.
- (c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.
- (d) What is the range of f for $t \geq 0$?

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

2009 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS

6. The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function f is defined by

$$f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2} \text{ for } x \neq 1 \text{ and } f(1) = 1. \text{ The function } f \text{ has derivatives of all orders at } x = 1.$$

- (a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.
 - (b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.
 - (c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).
 - (d) Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM